# ANSWERS TO SELECTED PROBLEMS IN STARR'S *General Equilibrium Theory: An Introduction*
# Chapter 2

**Selected Problems**

**Chapter 2,** Exercises 2.4, 2.9, 2.14.

**2.4.** Let $K = [0,1) \cup (1,2] \subset R$. Then $\bar{K} = [0, 2]$, a convex set. But K is non-convex since it does not include its midpoint. That is, $1 \notin K$.

**2.9.** Closed subsets of R: $S = \{ 0, 1, {}^1/_2, {}^1/_3, ..., {}^1/_\nu , ... \mid \text{for } \nu = 1, 2, 3, ... \}$. S is closed inasmuch as it contains its cluster points.

$T = [ 0, 1 ]$, the closed interval between 0 and 1.

Closed subsets of $R^N$: $A = \{ x = (x_1 , x_2, ..., x_N) \mid x_2 = x_3 = ...= x_N = 0, x_1 \in R\}$. A is the $x_1$ co-ordinate axis. A is a closed set since it includes all its cluster points.

$B = \{ x \mid x \in R^N , |x| \le 10 \}$. B is the closed ball of radius 10, centered at the origin.

**2.14.** • Show that $A \cap B$ is convex.

Let $x, y \in A \cap B$. Then $x, y \in$ A and B. Then by convexity of A and B we have that $\alpha x+(1-\alpha)y \in$ A and B. Then $\alpha x+(1-\alpha)y \in A \cap B$.

• Show that A+B is convex.

$x, y \in A+B$ means that there are $x^a, y^a \in A$, and $x^b, y^b \in B$ so that $x^a + x^b = x \in A+B$ and $y^a + y^b = y \in A+B$. Then

$\alpha x + (1-\alpha)y = \alpha x^a + \alpha x^b + (1-\alpha)y^a + (1-\alpha)y^b = \alpha x^a+(1-\alpha)y^a+\alpha x^b+(1-\alpha)y^b$. But $\alpha x^a+(1-\alpha)y^a \in A$, $\alpha x^b+(1-\alpha)y^b \in B$, by convexity of A and B. So $\alpha x + (1-\alpha)y \in A+B$.

• Show that $\bar{A}$ is convex.

Let $x, y \in \bar{A}$. We wish to show that $\alpha x+(1-\alpha)y \in \bar{A}$. The only difficulty arises if x or $y \notin A$. Suppose $x \notin A$. But then x is the limit of a sequence $x^\nu \in \bar{A}$. Then the sequence $\alpha x^\nu+(1-\alpha)y \in \bar{A}$ and approaches $\alpha x+(1-\alpha)y$ as its limit, so $\alpha x+(1-\alpha)y \in \bar{A}$.